breakdown shifts to the path IV. This pattern of breakdown, however, changes completely for $d/b = 0.5$. For small values of the septa width s/a, the P_m/λ_c^2 now corresponds to the breakdown along the path V and decreases rapidly. For $s/a \simeq 0.4$ the breakdown shifts to the gap edges (path II) and P_m/λ_c^2 drops sharply. Then for $s/a > 0.4$, the electric field at the septa edges (path IV) becomes the dominant factor in determining the breakdown power level. Whatever be the region of breakdown in the DLSG, P_m/λ_c^2 increases rapidly with increasing gap width d/b for $s/a < 0.5$. The power level also decreases monotonically with increasing s/a—very slowly when d/b is small and more rapidly for larger values of d/b.

IV. Conclusions

We have reported earlier that a Double L-Septa Guide has better cut-off and bandwidth characteristics than a Double T-Septa Guide [9], [10]. We have now extended the theoretical study of the DLSG by calculating the attenuation characteristics, septa-gap impedance and power handling capability of the dominant TE mode of the guide. The design data are presented for a wide range of septa parameters. These characteristics compare favorably with those of the ridged and T-septa guides and, together with the superior bandwidth, should make the DLSG a useful broadband transmission medium.

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On the Use of Levin's T-Transform in Accelerating the Summation of Series Representing the Free-Space Periodic Green's Functions

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***Abstract*—The Levin's t-transform is shown to accelerate the summation of slowly converging series. This is illustrated by application of the transform to the series representing the free-space periodic Green's functions involving a single and double infinite summation. Numerical results indicate that the transform converges rapidly than a direct summation of the series. Thus, it provides considerable savings in computation time.**

I. Introduction

The series representing the free-space periodic Green's function converges very slowly. This slow convergence results in making the analysis of periodic structures computationally expensive. In order to improve the efficiency of the codes, it is essential to employ methods to enhance the convergence of the Green's function series. Recently, a number of investigators [1]–[3] have used methods to reduce the computation time by a considerable amount. In this work, we show that the use of Levin's t-transform [4] is able to accelerate the summation of slowly converging series involving a single and double infinite summation. The primary advantage of the transform is that it is relatively free of roundoff errors in the computation of higher order iterates. In addition to this the transform can be applied to any slowly converging series without performing any analytical work prior to its application. The transform is outlined in Section II with an illustrative example. In Section III, the free-space periodic Green's functions are given. The numerical results and conclusion are presented in Sections IV and V, respectively.

II. Levin 's t-Transform

Let S_n be the partial sum of n terms of a series such that $S_n \rightarrow S$ as $n \rightarrow \infty$, where S is the sum of the series. The Levin's t-transform may be computed as follows:

$$t_k^{(n)} = \frac{\displaystyle\sum_{i=0}^{k} (-1)^i \binom{k}{i} \left(\frac{n+i}{n+k}\right)^{(k-1)} \left(\frac{S_{n+i}}{S_{n+i+1} - S_{n+i}}\right)}{\displaystyle\sum_{i=0}^{k} (-1)^i \binom{k}{i} \left(\frac{n+i}{n+k}\right)^{(k-1)} \left(\frac{1}{S_{n+i+1} - S_{n+i}}\right)}, \quad k = 0, 1, 2, \cdots \qquad (1)$$

The kth order transform, $t_k^{(n)}$ or $t_k(S_n)$, gives an estimate of the sum, S, of the series. An inherent advantage of the t-transform is that the higher order iterates are computed from the partial sums. Hence, the accuracy to which the partial sums are computed can be preserved in the transform computations. This keeps the transform relatively immune to roundoff errors in comparison to Shanks' transform [5] in which higher order iterates are computed from the lower orders resulting in severe loss of significant digits due to accumulation of roundoff errors [6]. The transform can be illustrated by applying it

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TABLE I
APPLICATION OF LEVIN'S t-TRANSFORM TO THE SERIES FOR ln 3

n	$t_0^{(n)} = S_n$	$t_1^{(n)}$	$t_2^{(n)}$	$t_3^{(n)}$	$t_4^{(n)}$	$t_5^{(n)}$	$t_6^{(n)}$
1	2.0000000	1.1428572	1.0909091	1.0994152	1.0985777	1.0986093	1.0986133
2	0.0000000	1.0666667	1.1014494	1.0984458	1.0986129	1.0986135	1.0986127
3	2.6666667	1.1282053	1.0970465	1.0986667	1.0986135	1.0986123	
4	−1.3333333	1.0666668	1.0997245	1.0985881	1.0986122		
5	5.0666671	1.1368420	1.0976750	1.0986261			
6	−5.5999999	1.0493511	1.0995084				
7	12.6857157	1.1657153					
8	−19.3142853						
9	37.5746040						

to the divergent Taylor series for $\ln(1+x)$ when $x = 2$:

$$\ln 3 = -\sum_{p=1}^{+\infty} \frac{(-2)^p}{p} \tag{2}$$

The result of applying the Levin's t-transform to the sequence of partial sums $S_1, S_2, \cdots, S_9$ is given in Table I. Although not shown in the table, the last entry is $t_7^{(n)} = 1.0986125$. The result is computed in single precision arithmetic and compared to the exact result of 1.0986123 it is accurate to seven digits.

III. PERIODIC GREEN'S FUNCTIONS

The Green's function for a one-dimensional array of line sources spaced d units apart along the y-axis is given by

$$G = \sum_{p=-\infty}^{+\infty} \frac{1}{4j} H_0^{(2)} \cdot (k[(x-x')^2 + (y-y'-pd)^2]^{1/2}) \tag{3}$$

where $H_0^{(2)}$ is the zeroth-order Hankel function of the second kind, k is the wavenumber of the medium, (x', y') is the location of the reference source and (x, y) locates the observation point. The spatial domain Green's function series in (3) converges very slowly for all combinations of source and observation points. Due to this reason the spectral domain representation of the Green's function is often employed in the numerical analysis of periodic structures with one-dimensional periodicity. The spectral domain Green's function is given by:

$$G = \sum_{p=-\infty}^{+\infty} \frac{1}{j2dk_{x_p}} e^{-jk_{x_p}|x-x'|} e^{-j2p\pi(y-y')/d} \tag{4}$$

where

$$k_{x_p} = \begin{cases} \sqrt{k^2 - (2p\pi/d)^2}, & k^2 > (2p\pi/d)^2 \\ -j\sqrt{(2p\pi/d)^2 - k^2}, & k^2 < (2p\pi/d)^2. \end{cases} \tag{5}$$

The spectral domain Green's function series converges rapidly for $x \neq x'$ ("off plane" case). This is due to the exponential factor which aids in the convergence. However, for $x = x'$ ("on plane" case), the series in (4) converges very slowly and may take anywhere from 10^4 to 10^6 terms to converge. This poses a significant problem since in a moment method solution repeated evaluations of the Green's function series are required. The slow convergence of this series can make the analysis process computationally expensive. Therefore, it is imperative to use some means to accelerate the convergence of the series.

The free-space periodic Green's function for a two-dimensional infinite array is given by:

$$G(\boldsymbol{r}) = \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \frac{1}{j2Ak_{zpq}} e^{-jk_{zpq}|z|} e^{-j\boldsymbol{k}_{tpq}\cdot\boldsymbol{r}} \tag{6}$$

where $\boldsymbol{r}$ is the location of the observation point and it is assumed without the loss of generality that the reference source is located at the origin. Also, A is the area of the unit cell and

$$k_{zpq} = \begin{cases} \sqrt{k^2 - |\boldsymbol{k}_{tpq}|^2}, & k > |\boldsymbol{k}_{tpq}| \\ -j\sqrt{|\boldsymbol{k}_{tpq}|^2 - k^2}, & k < |\boldsymbol{k}_{tpq}| \end{cases} \tag{7}$$

$$\boldsymbol{k}_{tpq} = (p + p_0)\boldsymbol{k}_1 + (q + q_0)\boldsymbol{k}_2 \tag{8}$$

$$\boldsymbol{k}_1 = (2\pi/D_x)\hat{x}, \qquad \boldsymbol{k}_2 = (2\pi/D_y)\hat{y} \tag{9}$$

$\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are reciprocal lattice base vectors defined in (9) for a rectangular lattice, D_x, D_y are the periodicities in the x, y directions, respectively, p_0 and q_0 are the interelement phase shift constants, and k is the free-space wavenumber. The series has the slowest convergence as the observation point approaches the source plane, i.e., as $z \to 0$ ("on plane" case).

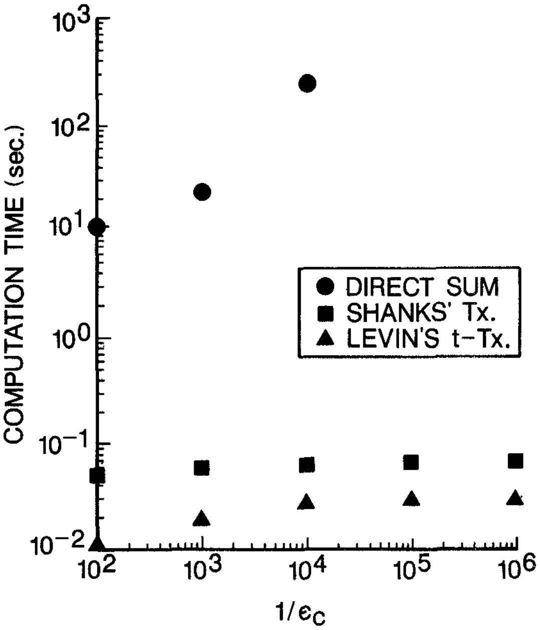

Fig. 1. Computation time versus $1/\epsilon_c$ for the spatial domain Green's function series in (3) for $(x, y) = (0.4\lambda, 0.3\lambda), d = 0.6\lambda$, and $\lambda = 0.8m$.

IV. NUMERICAL RESULTS

In this section, we present the results of applying the t-transform to the partial sums of the series given in (3), (4) and (6). To provide a comparison the results from Shanks' transform and direct summation of the series are also given. In order to stop the iteration process, the convergence criterion used in [1] is employed. In this criterion, ϵ_c is the convergence factor and it is provided as part of the input specification. The reference source is taken to be at the origin.

Figs. 1–2 show the computation time (on a VAX 6350) vs. $1/\epsilon_c$ for the one-dimensional spatial and spectral domain Green's function series, respectively. In each case the t-transform converges to machine precision in 0.03 seconds. In comparison, the direct sum does not converge and takes several minutes to run. The computation time vs. $1/\epsilon_c$ for the two-dimensional Green's function series is shown in Fig. 3. For $\epsilon_c = 10^{-5}$, the t-transform converges to a high degree of accuracy within 0.2 seconds whereas the direct sum takes 250 seconds and does not give comparable accuracy.

V. CONCLUSION

The t-transform significantly accelerates the convergence of the series representing the one- and two-dimensional periodic Green's functions. The numerical results confirm that the t-transform is effective even in the "on plane" case when the spectral domain series

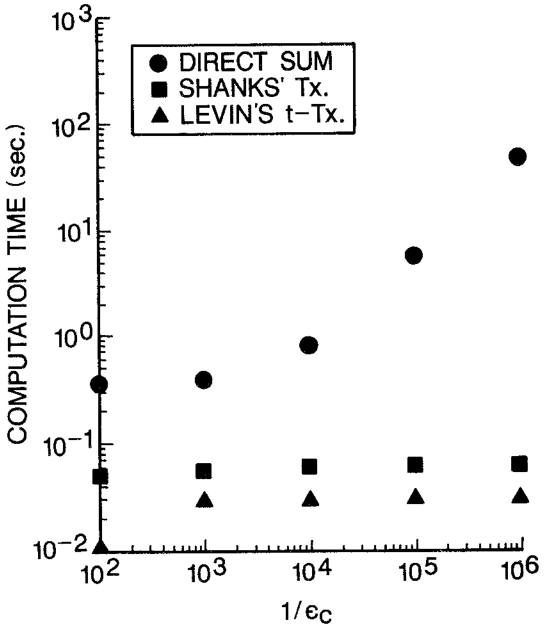

Fig. 2. Computation time versus $1/\epsilon_c$ for the spatial domain Green's function series in (4) for $(x, y) = (0.0\lambda, 0.6\lambda), d = 1.2\lambda$, and $\lambda = 1.0m$.

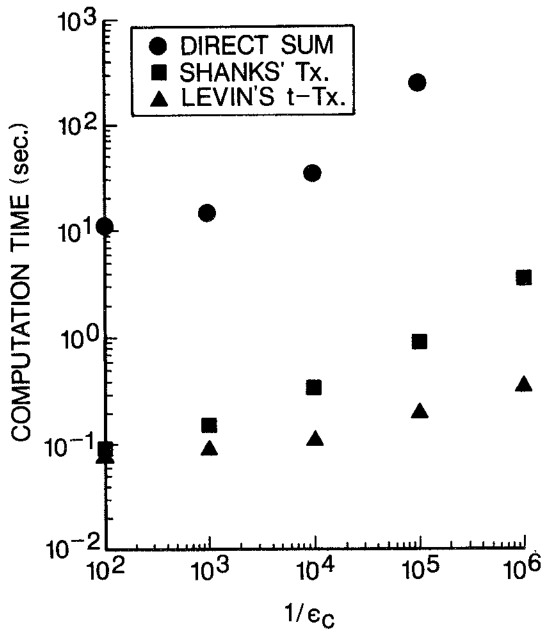

Fig. 3. Computation time versus $1/\epsilon_c$ for the spatial domain Green's function series in (6) for $(x, y, z) = (0.6\lambda, 0.6\lambda, 0.0\lambda), D_x = D_y = 0.75\lambda$, $p_0 = q_0 = 0, \lambda = 1.0m$.

have the slowest convergence. This results in a considerable savings in computation time.

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Computer-Aided Design of a Singly-Matched (S-M) Multiplexer with a Common Junction

Ji-An Gong and Wai-Kai Chen

Abstract—**In this paper, the formulas based on scattering parameters are presented for the design of a multiplexer composed of $n - 1$ channel equalizers connected either in parallel or in series at a common junction with a 1-Ω resistive generator and $n - 1$ channel complex loads. It is known as the singly-matched (S-M) multiplexer. A new two-stage computer-aided design approach is developed for the S-M multiplexer. A design example of a three-channel singly-matched multiplexer including the designs of three individual S-M channel equalizers is given to demonstrate the approach.**

I. Introduction

Many multiplexer design techniques have been developed [1]–[3], but none of them considers the complex load impedances. With recent developments in solid-state technology, a pure resistive model of the load is no longer an adequate representation.

In this paper, a multiplexer configuration consists of $n - 1$ channel equalizers connected in either parallel or series at a common junction. All the loads of the multiplexer are assumed to be complex, and the generator is assumed to be connected in series with a 1-Ω resistor. A multiplexer with a resistive source and complex loads is called a singly-matched (S-M) multiplexer. Formulas are presented for either the parallel or series configuration with a common junction. A new two-stage computer-aided procedure is developed for their design. At the first stage, each channel equalizer is designed to be a singly-matched so that the transfer of power from the 1-Ω resistive generator to the channel complex load is maximized over a prescribed channel frequency band. This is known as a single broad-band matching problem for which many papers have been published [4]–[6]. Since the ladder structure is attractive not only from a practical viewpoint, but also effective as an equalizer in most applications, in this paper each channel equalizer is assumed to be a two-port lossless ladder network. The S-M channel equalizer is realized by optimization matching technique, thereby making it easier to design a S-M equalizer having different types of responses (Chebyshev or elliptic) and various passbands (lowpass, bandpass, or highpass). At the second stage, by using the formulas and existing optimization techniques, all the element values in the multiplexer are modified until a good match is achieved at the common input port over the entire transmission band. Since all the designs are

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